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| Mathematics Design Collaborative |
| State of Georgia Department of Education |



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| **Appropriate Scaling for Linear Graphs, Understanding Constraints** |
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| ***The Largest Loser*** |
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| **INTRODUCTION TO THIS FORMATIVE ASSESSMENT LESSON** |
| **MATHEMATICAL GOALS** |
| This lesson unit is intended to help you assess how well students are able to: |
| * Utilize what they already know about linear equations in the context of different graphs. * Reasoning quantitatively, choose and interpret the appropriate scale and rate of change from graphs. * Understand Constraints upon graphs in given contexts and make sense of graph problems with differently-defined axes of measure. * Reason abstractly and compare graphs of linear equations with different scales of measure. |
| **GEORGIA STANDARDS OF EXCELLENCE** |
| This lesson involves mathematical content in the standards from across the grades, with emphasis on:: |
| **MGSE9-12 N.Q.1** **Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:**   1. **Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;** 2. **Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);** 3. **Use units within multi-step problems and formulas; interpret units of input and resulting units of output.**   **MGSE9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers’ precision is limited to the precision of the data given.***  **MGSE9-12.A.CED.2 Create linear, ~~quadratic,~~ and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which A = P(1 + r/n)nt has multiple variables.)** |
| * SMP1. Make sense of problems and persevere in solving them. * SMP2. Reason abstractly and quantitatively. * SMP4. Model with mathematics * SMP5. Use appropriate tools strategically. * SMP6. Attend to precision. |
| **INTRODUCTION** |
| This lesson is structured in the following way: |
| Before the Lesson,  Have students do the Pre-Assessment task in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson. |
| At the Start of the Lesson,  Have students work independently at the beginning of the lesson. They complete the chart that shows the sliding scale for Barbie’s weight loss. |
| During the Lesson,  Group students in pairs by common errors found in Pre-Assessment. Students are given **precut** card sets A and B. Students should match one card from A with one card from B. |
| After the Whole-Group Class Discussion,  Give students a second blank copy of their tasks back. Return to the students their original assessment task if you choose. |

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| **MATERIALS REQUIRED** | | | | | | |
| Each individual student will need: | | | | | | |
| * **Precut** card sets of A and B * Poster Paper * Markers * Glue Sticks or Tape | | | | | | |
| **TEACHER PREP REQUIRED** | | | | | | |
| Teacher, be advised that prior to the lesson, the following preparations/copies will need to be made: | | | | | | |
| * Copies of Materials * **Precut** card sets of A and B | | | | | | |
| **TIME NEEDED:** | | | | | | |
| For Pre-Assessment: 15 min | For Lesson: 55 min | For Post: 20 min |  |  |
| **FRAMING FOR THE TEACHER:** | | | | | |
| This FAL should be done at the end of the unit. Failure to read and interpret graphs for what they actually say is a common error students make. Note that the activity itself (slope, y-intercept, graphing lines, equations of lines) is really the focus of this lesson, even though their existence adds meat to the lesson. This FAL allows students to use some linear equation skills as a tool for investigating relationships between quantities rather than a free-standing skill. | | | | | |
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| **FRAMING FOR THE KIDS:** | | | | | |
| Say to the students: | | | | | |
| *This activity will take about \_\_2\_\_days for us to complete.* | | | | | |
| *The reason we are doing this is to be sure that you understand: how to read and interpret relationships between quantities before we move on to a new idea.* | | | | | |
| *You will have a chance to work with a partner to correct any misconceptions that you may have. After the partner work, you will be able to show me what you have learned!* | | | | | |

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| **PRE-ASSESSMENT BEFORE THE LESSON** |
| **ASSESSMENT TASK: The Largest Loser** |
| Time This Should Take: 15 minutes |

Have the students do this task in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will them be able to target your help more effectively in the follow-up lesson.



Give each student a copy of the Pre-Assessment: The Largest Loser

Briefly introduce the task and help the class to understand the problem and its context.

*Spend 15 minutes working individually on this task. Read through the task and try to answer it as carefully as you can. Show all your work so that I can understand your reasoning. Don’t worry if you can’t complete everything. There will be a lesson that should help you understand these concepts better. Your goal is to be able to confidently answer questions similar to these by the end of the next lesson.*

Students should do their best to answer these questions, without teacher assistance. It is important that students are allowed to answer the questions on their own so that the results show what students truly do not understand.

Students should not worry too much if they cannot understand nor do everything on the pre-assessment, because in the next lesson they will engage in a task which is designed to help them. Explain to students that by the end of the next lesson, they should expect to be able to answer questions such as these confidently.

This is their goal.

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| **COLLABORATION TIME/READING STUDENTS RESPONSES** | |
| **You Will Not “Grade” These!** | |
| Collect students’: responses to the task. It is helpful to read students’ responses with colleagues who are also analyzing student work. Make notes (on your own paper, not on their pre-assessment) about what their work reveals about their current levels of understanding, and their approaches to the task. You will find that the misconceptions reveal themselves and often take similar paths from one student to another, and even from one teacher to another. Some misconceptions seem to arise very organically in students’ thinking. Pair students in the same classes with other students who have similar misconceptions. This will help you to address the issues in fewer steps, since they’ll be together. (Note: pairs are better than larger groups for FAL’s because both must participate in order to discuss!) | |
| You will begin to construct Socrates-style questions to try and elicit understanding from students. We suggest you write a list of your own questions; however some guiding questions and prompts are also listed below as a jumping-off point. | |
| **GUIDING QUESTIONS** | |
| ***COMMON ISSUES*** | ***SUGGESTED QUESTIONS AND PROMPTS*** |
| Student Was Unable to Begin Assessment  *Example: “I don’t know who won,” “You can’t tell who lost the most weight, because you can’t see what their original weights were.”* | * *If you pointed to a location on Mike’s graph, what would it mean in terms of the graph labels?* * *If you just pretended both Mike & Ramon both weighed 300 pounds at start, could you make chart that showed how much they weighed at start, at 2 weeks, at 4 wks, at 6 wks, at 8 wks, & at 10 wks?* |
| Student did not note that the y-axes on the two graphs represented different measures.  *Example: “Mike Lost 6 pounds in 10 weeks, and Ramon Lost 5 pounds in 10 weeks”* | * *What differences do you see in labels on two graphs?* * *Look at Mike’s graph. How long did it take him to lose two pounds? What y-value shows you that?* * *Ramon wanted to lose 10 pounds, right? So what y-value shows when he already lost 2 pounds? How long did that take Ramon?* |
| Student read Mike’s “Pounds Lost” as equivalent to Ramon’s “Pounds Left to Lose.”  *Example: “Mike’s weight went up, but Ramon’s weight went down, and so Ramon lost more weight.”* | * *Does it mean the same thing for your weight to decrease and for your goal to decrease?* |
| Student did not identify the two contestants’ weight loss as equal over the given number of weeks.  *Example: Student does not realize that Mike losing 6 pounds over 10 weeks is the same as Ramon’s goal moving from 10 pounds to four pounds in the same amount of time means that he also lost 6 pounds.* | * *If I spent $25 less this week than last week, can you think of another way to say that?* * *If I already ran 20 yards of 100-yard dash, how many yards do I have left to run toward my 100-yard goal?* * *If I moved 10 lbs away from my weight loss goal, what does that mean I did?* * *Ramon’s goal was to lose 10 lbs, and right here (point to 4 wks) it shows he has 8 lbs left. Did he get 2 lbs closer to his goal or 2 lbs farther away from his goal?* * *If Ramon is 2 lbs closer to goal, how much was lost?* |
| Student unable to calculate correct rate of weight loss.  *Example: Student mentioned weight loss/decreasing distance to weight loss goal but did not mention specific quantities* | * *Can you think of a mathematical term that means “rate of change?”* * *How do you calculate slope of a line?* |
| Student was unable to explain his or her reasoning for deciding a winner in the Largest Loser contest  *Example: Student says “just because” or fails to make a case that is backed up by numerical calculations*. | * *How would you tell someone you are right about your decision, if they insist that you are wrong?* * *How can you demonstrate numerically with data which “loser” is the winner?* |
| Student Was Unable to Begin Assessment  *Example: “I don’t know who won,” “You can’t tell who lost the most weight, because you can’t see what their original weights were.”* | * *If you pointed to a location on Mike’s graph, what would it mean in terms of the graph labels?* * *If you just pretended both Mike and Ramon both weighed 300 lbs at start, could you make a chart that showed how much they weighed at start, at 2 wks, at 4 wks, at 6 wks, at 8 wks, and at 10 wks?* |

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| **LESSON DAY** | | |
| **SUGGESTED LESSON OUTLINE:** | | |
| **Part 1: Whole-Class Introduction:** | **Time to Allot: ( 10 minutes)** | |
| If you have a short lesson, or you find the lesson is progressing at a slower pace than anticipated, then we suggest you end the lesson after the first collaborative activity and continue in a second lesson.  **Whole-class interactive introduction:**   |  | | --- | |  | | Students complete the activity independently at the beginning of the lesson. They complete the chart that shows the sliding scale for Barbie’s weight loss. | | | |
| Suggested Prompts: | | |
| * *Questions to ask:* * *What does the y-intercept of the original graph represent? (i.e. what is Barbie’s goal?)* * *What is the relationship between the pounds left to lose and the weight lost? How can you figure out weight lost if you know how many pounds Barbie has left to lose?* * *Will Barbie meet her goal? If so, when?* * *How did you figure out the values beyond the parts of the graph you can see (if Barbie continues on this same track)?* * *What equation can you write that would show Barbie’s weight loss?* | | |
| **Part 2: Collaborative Activity:** | **Time to Allot: ( 40 minutes)** | |
| Put students into their pairs according to your analysis of student errors. | | |
| Do/Say the Following:  Collaborative Activity Part 1: (\_20\_ Minutes)   * Group students in pairs by common errors found in Formative Assessment comparing Mike and Ramon’s weight loss. * Students are given card sets A and B, already cut apart. * Students should match one card from A with one card from B. | | |
| Collaborative Activity Part 2: Sharing or Presenting (20 minutes)  |  | | --- | | * Students will put the contestants in order from most successful in losing weight to least successful. | | * These contestants will be ranked by name and graph letter. * The students will demonstrate the rate of change for each graph in terms of weight loss, not in terms of amount of weight left to lose. | | | |
| During the Collaborative Activity, the Teacher has 3 tasks:   * Circulate to students’ whose errors you noted from the pre-assessment and support their reasoning with your guiding questions. * Circulate to other students also to support their reason in the same way. * Make a note of student approaches for the summary (plenary discussion). Some students have interesting and novel solutions! | | |
| **Part 3: Plenary (Summary) Discussion:** | **Time to Allot: ( 20 minutes)** | |
| Gather students together, share solutions. Discussion prompts should be made up of your original guiding questions and notes about student approaches. Some other discussion prompts are listed below: | | |
| NOTE: *“Scribing” helps to increase student buy-in and participation. When a student answers your question, write the student’s name on the board and scribe his/her response quickly. You will find that students volunteer more often when they know you will scribe their responses – this practice will keep the discussions lively and active!*   * *Students will present their charts.* * *So who was the winner?* * *How can you be sure that your equation is correct for any of these situations?* * *What do you notice about the rate of change for those contestants who were at the top?* * *What do you notice about the rates of change for those contestants who were at the bottom?* * *What did the horizontal line represent?* * *What would a negative y value represent?* * *If the y-intercept was negative 30, what would that mean?* * *What would you infer if the y-intercept were 80?* * *What you tell George if his weight loss equation were y = 2x + 45?* | | |
| **Part 4: Improving Solutions to the Assessment Task** | | **Time to Allot: ( 20 minutes)** |
| The Shell MAP Centre advises handing students their original assessment tasks back to guide their responses to their new Post-Assessment (which is sometimes the exact same as the Pre-Assessment). In practice, some teachers find that students mindlessly transfer incorrect answers from their Pre- to their Post-Assessment, assuming that no “X” mark means that it must have been right. . Until students become accustomed to UNGRADED FORMATIVE assessments, they may naturally do this. Teachers often report success by handing students a list of the guiding questions to keep in mind while they improve their solutions.  Practice will make perfect, and teachers should do what makes them most comfortable with their students/finds misconceptions! | | |

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| **PRE-ASSESSMENT (Answer Key)** |
| **ASSESSMENT TASK: The Largest Loser Graphs** |
| The Largest Loser Graphs  Mike and Ramon are both contestants in a weight loss contest. Part of the contest requires that they graph their progress. See their charts, below.   1. You are one of the contest judges. Look carefully at the two graphs, and make a very detailed mathematical case, in writing, below for which contestant you think should be named the winner that you think would be convincing to the other judges. *Looking at the graphs, it appears at first glance that Ramon lost more weight because his graph is going downhill. But if you look at the y axis labels, they are different. So just looking at the graphs isn’t enough to decide.*     *Looking at a chart may be a good way to approach this; also it may be good to make up a common starting weight for both. If Mike and Ramon both started off at 300 pounds….*     1. One of your fellow judges wants to know how fast each contestant lost weight. How could you explain how he can calculate each contestant’s rate of weight loss?   *The chart above shows that Mike and Ramon actually lost the same amount of weight. They both win!* ***High Fives and Ice Cream Sundaes all around****!!! ☺* |

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| **Collaborative Activity (Answer Key)** |

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| **PRE-ASSESSMENT** |
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| The Largest Loser Graphs  Mike and Ramon are both contestants in a weight loss contest. Part of the contest requires that they graph their progress. See their charts, below.   1. You are one of the contest judges. Look carefully at the two graphs, and make a case for which contestant you think should be named the winner that you think would be convincing to the other judges.      1. One of your fellow judges wants to know how fast each contestant lost weight. How could you explain how he can calculate each contestant’s rate of weight loss? |

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| **COLLABORATIVE ACTIVITY** |
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| *Card Set A: Contestants*   |  |  | | --- | --- | | **Contestant A** | **Contestant B** | | **Contestant C** | **Contestant D** | | **Contestant E** | **Contestant F** | | **Contestant G** | **Contestant H** | |

# *Card Set B: Situations*

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| **2**  **Mabel lost ½ pound each week.** | **7**  **Gladys lost 1 pound each week.** |
| **6**  **Willomena gained 5 pounds each week.** | **3**  **Emmy gained ½ pound each week.** |
| **1**  **Hedwig gained 2 pounds each week.** | **4**  **Tricia lost 2 pounds each week.** |
| **8**  **Ethel lost 3 pounds each week.** | **5**  **Hazel didn’t lose any weight.** |

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| **Lesson Day Warm-Up** |
| Name of Assessment Task: |
| *If the graph below shows Barbie’s progress toward her weight loss goal, complete the TABLE below to show her progress. Then complete the chart on the right.*     |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | Week Number | Pounds Left to Lose | Weight Lost | Week Number | Pounds Left to Lose | Weight Lost | | **1** |  |  | **6** |  |  | | **2** |  |  | **7** |  |  | | **3** |  |  | **8** |  |  | | **4** |  |  | **9** |  |  | | **5** |  |  | **10** |  |  | |

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| **Collaborative Activity Instructions:** |
| *Collaborative Activity Instructions: The Largest Loser Contestants were asked to make a chart of their progress over the last ten weeks. Since they were given no more instructions than this, they made different types of graphs.*   1. You have been grouped in pairs. 2. You are given card sets A (each contestants’ graphs) and B (each contestants’ results written out verbally), already cut apart. 3. Read the graphs and situations very critically and very carefully. You and your partner should match each graph from card set A from each situation from card set B. Discuss to ensure that you both agree. 4. Once you are sure that you have completed the matching correctly, rank all of these contestants from “WINNER” (most weight lost) all the way down to 8th place (least weight lost). 5. Grab a sheet of chart paper and glue them, in order, paired. 6. Beside each pair, calculate (and write): RATE OF WEIGHT LOST EACH WEEK. 7. If you finish early, beside each pair write the appropriate LINEAR EQUATION. Be prepared to justify your answers and to discuss. |